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CONSTRUCTING OF A CANAL SURFACE, REFERRED TO CURVATURE LINES, AS A SET OF CIRCLES OF THE CURVATURE OF A CONICAL HELIX

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We consider the constructing of a canal surface, referred to the curvature lines in the system of an accompanying three-edge of a conical helix. A cyclic framework of the curvature lines of the canal surface was formed by the curvature circles of a conical helix. Parametric equations of the canal surface were obtained and visualized.

Keywords: *canal surface, accompanying three-edge of Frenet, line of centers, the first quadratic form of the surface*

The task of referring of the surface to the curvature lines is an important geometrical problem caused by the convenience of using this parametrization during the research of interaction of environment and surfaces. The advantage of this analytical definition of the surface is a particularly simple form of the first and second quadratic forms at any point of the surface. A surfaces class that can be referred to the curvature lines is very limited and includes Monge carved surfaces, Dupin cyclides, Joachimsthal surfaces.

Among the constructive ways of referring of the canal surfaces to the curvature lines we use the method in which the cyclic framework of a canal surface is formed by the movement of a circle in the system of a Frenet accompanying three-edge of a spatial guide curve [2, 6]. Then, the problem of referring to the curvature lines of the canal surface leads to a Riccati differential equation, which is generally not integrated in quadratures [4].

Purpose of the research – to determine the conditions for referring of canal surface to the curvature lines, to find parametric equations of this surface and to visualise it.

Materials and methods of the research. Lets define a spatial line f by vector equations in its arc length function:

$$\overline{r} = \overline{r}(s) = x(s) \cdot \overline{i} + y(s) \cdot \overline{j} + z(s) \cdot \overline{k}, \qquad (1)$$

where $\overline{i}$; $\overline{j}$; $\overline{k}$ – the unit vectors of a fixed coordinate system $Oxyz$. Each point of a space curve (1) is corresponded to its curvature circle, which will be located in a osculating plane of the Frenet three-edge. A circle curvature radius is: $R = R(s) = \dfrac{1}{k(s)}$, where $k = k(s)$ – curvature of a space curve f. Then, during the motion of the Frenet three-edge of along a spatial guide curve f, a cyclic frame of curvature circles form a family of curvature lines of the canal surface [4]. Vector equation of the canal surface formed by the set of curvature circles of the space curve f defined by the equation (1) has the form [4]:

$$\overline{R}(v,s) = \overline{r}(s) + \overline{\tau} \cdot \frac{1}{k(s)} \cdot \cos v + \overline{n} \cdot \frac{1}{k(s)} \cdot (1 + \sin v), \qquad (2)$$

where $v -$ an independent variable, $k = k(s)$ – a space curve curvature f.

To find family lines orthogonal to the set of circular frame circles, we must solve the differential equation of the orthogonal trajectories. This equation for the canal surface defined by a vector equation (2) has the form [4]:

$$\frac{dv}{ds} = -\left(\left(\frac{1}{k(s)} \right)'_s \cdot \cos v + 1 \right) \cdot k(s). \qquad (3)$$

The differential equation solution (3) is an analytic condition of referring of canal surface, the cyclic framework of which is formed by of curvature circles of the space curve f to the curvature lines.

Research results. The differential equation (3) is referred to the differential equations wich are insoluble in the derivative [5] of the unknown function $v = v(s)$.

Let’s replace:

$$t(v) = tg\frac{v}{2}. \tag{4}$$

Location: $v = 2 \cdot arctg(t); \ dv = \frac{2}{1+t^2} \cdot dt.$

The substitution of the last equation in the differential equation (3) after simplification converts it into an incomplete Riccati differential equation [5]:

$$\frac{dt}{ds} = -\left(t^2(s) \cdot \frac{k^2(s) + k'_s}{2 \cdot k(s)} + \frac{k^2(s) - k'_s}{2 \cdot k(s)} \right), \tag{5}$$

where $t = t(s) -$ an unknown function, $k(s) -$ a defined curvature of a space guide curve (1).

A differential equation (5) is generally not integrated in quadratures, but can be reduced to a differential equation with separable variables for individual functions $k(s)$.

If $k = const,$ then we obtain a condition for referring of tubular surface (cyclic framework of the circles of constant radius $R = \frac{1}{k}$) to the curvature lines. An analytical description of this surface detail by in the [1].

If $\frac{k^2(s) + k'_s}{2 \cdot k(s)} = 0,$ the differential equation $k^2(s) + k'_s = 0$ has a particular solution:

$$k(s) = \frac{1}{s}. \tag{6}$$

Substitution of curvature (6) in the Riccati differential equation (5) reduces it to the equation: $\frac{dt}{ds} = -\frac{1}{s},$ the general solution of which is the function:

$$t(s) = u - \ln s. \tag{7}$$

In the expression (7), variable u is an arbitrary constant of integration, which will be a new variable in parametric equations of the surface of the variable $v.$ Taking into account the change (4) to expression (7), we obtain analytical conditions of referring of canal surface to the lines of curvature as a set of circles of curvature of the space curve (one of the natural space curve equations has a form $k(s)=\frac{1}{s}$):

$$v(s)=2\cdot arctg(u-\ln s). \qquad (8)$$

If in the differential equation (5): $a\cdot\frac{k^2(s)+k_s'}{2\cdot k(s)}=\frac{k^2(s)-k_s'}{2\cdot k(s)}$, where a – a constant, so the curvature of the guide curve is $k(s)=\frac{a+1}{(a-1)\cdot s}$. We received back proportionality which is similar to the function (6), so the analytical condition of surface referring to the curvature lines will not be found.

Let's consider a conical helix slope, defined by the dependence (6) of the curvature on the arc length and the angle $\beta=const$, which forms a tangent to the guide curve and the horizontal plane, as a spatial guide curve (1). The parametric equations of this curve is found by the formula [3]:

$$\begin{aligned} x&=\cos\beta\cdot\int\cos\left(\frac{1}{\cos\beta}\int kds\right)ds;\\ y&=\cos\beta\cdot\int\sin\left(\frac{1}{\cos\beta}\int kds\right)ds;\\ z&=s\cdot\sin\beta. \end{aligned} \qquad (9)$$

Substituting (6) into (9), we obtain the parametric equations of the conical helix:

$$\begin{aligned} x(s)&=\frac{s\cdot\cos^2\beta}{1+\cos^2\beta}\cdot(\cos\beta\cdot\cos(\gamma(s))+\sin(\gamma(s)));\\ y(s)&=\frac{s\cdot\cos^2\beta}{1+\cos^2\beta}\cdot(\cos\beta\cdot\sin(\gamma(s))-\cos(\gamma(s)));\\ z(s)&=s\sin\beta, \end{aligned} \qquad (10)$$

where $\gamma(s)=\frac{\ln(s)}{\cos\beta}$.

According to the methods described in the work [3], we find the parametric equations that match the vector equation (2) of the canal surface formed by the set of circles of curvature conical helix:

$$X(v,s)=\frac{s\cdot\cos^2\beta}{1+\cos^2\beta}\cdot(\cos\beta\cos\gamma+\sin\gamma)+$$
$$+s\cdot(\cos\beta\cos v\cos\gamma-(1+\sin v)\sin\gamma);$$
$$Y(v,s)=\frac{s\cdot\cos^2\beta}{1+\cos^2\beta}\cdot(\cos\beta\sin\gamma-\cos\gamma)+ \tag{11}$$
$$+s\cdot(\cos\beta\cos v\sin\gamma-(1+\sin v)\cos\gamma);$$
$$Z(v,s)=s\sin\beta\cdot(1+\cos v);$$

where $\gamma(s)=\dfrac{\ln(s)}{\cos\beta}$.

The canal surface defined by the equations (11), has only one family of curvature lines –a cyclic framework. If we substitute the condition (8) into the equation (11), we get parametric equations $X(u,s), Y(u,s), Z(u,s)$ of the canal surface, referred to the curvature lines as a set of curvature circles of the conical helix (Fig. 1).

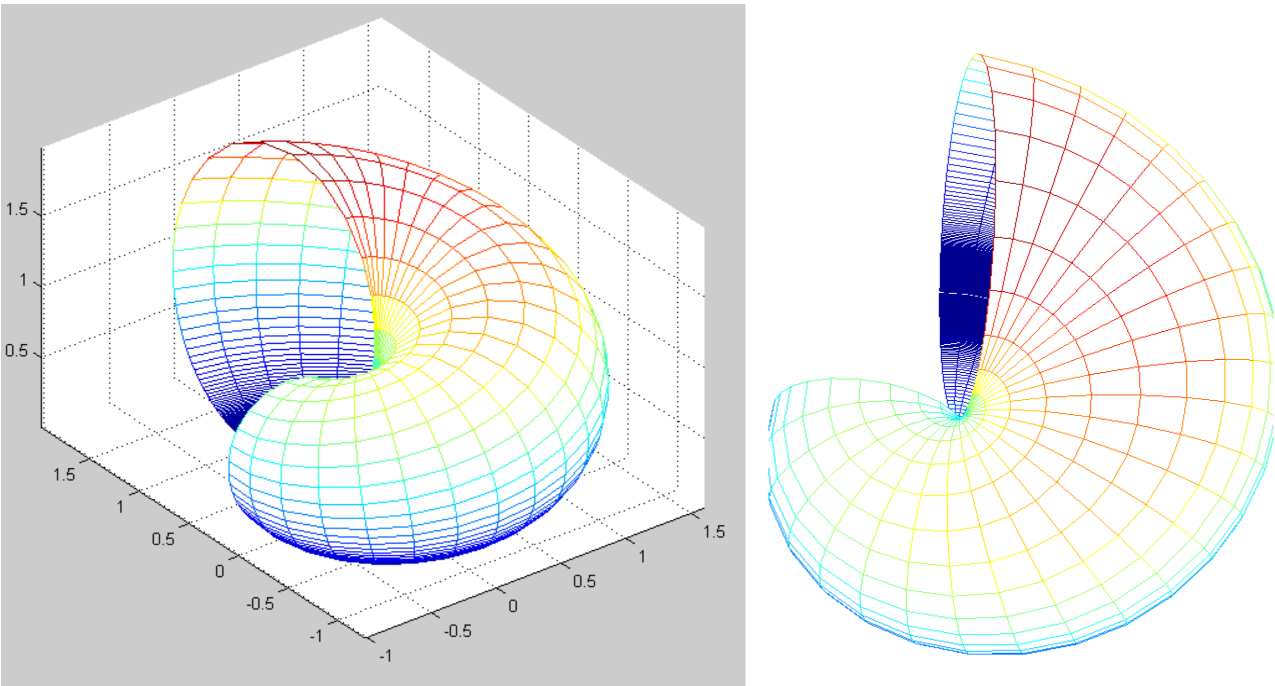

Fig.1. Canal surface, referred to the curvature lines and its horizontal projection.

Conclusions

1. Using of a spatial guide curve (a conical helix), the curvature of which is $k(s) = \frac{1}{s}$ (where s – the length of the curve arc), makes it possible to find an analytic condition (8) of referring of canal surface to the curvature lines.

2. We have found the parametric equations of the canal surface, referred to the curvature lines, and visualised it.

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КОНСТРУЮВАННЯ КАНАЛОВОЇ ПОВЕРХНІ, ВІДНЕСЕНОЇ ДО ЛІНІЙ КРИВИНИ, ЯК МНОЖИНИ КІЛ КРИВИНИ КОНІЧНОЇ ГВИНТОВОЇ ЛІНІЇ

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Розглянуто конструювання каналової поверхні, віднесеної до ліній кривини у системі супровідного тригранника конічної гвинтової лінії. Циклічний каркас ліній кривини каналової поверхні утворено за допомогою кіл кривини конічної гвинтової лінії. Отримано параметричні рівняння каналової поверхні, здійснено її візуалізацію.

Ключові слова: *каналова поверхня, супровідний тригранник Френе, лінія центрів, перша квадратична форма поверхні*

КОНСТРУИРОВАНИЕ КАНАЛОВОЙ ПОВЕРХНОСТИ, ОТНЕСЕННОЙ К ЛИНИЯМ КРИВИЗНЫ, КАК МНОЖЕСТВА ОКРУЖНОСТЕЙ КРИВИЗНЫ КОНИЧЕСКОЙ ВИНТОВОЙ ЛИНИИ

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Рассмотрено конструирование каналовой поверхности, отнесенной к линиям кривизны в системе сопровождающего трёхгранника конической винтовой линии. Циклический каркас линий кривизны каналовой поверхности образован с помощью окружностей кривизны конической винтовой линии. Получены параметрические уравнения каналовой поверхности, осуществлена её визуализация.

Ключевые слова: *каналовая поверхность, сопровождающий трёхгранник Френе, линия центров, первая квадратичная форма поверхности*